

Comparative Analysis of Chinese and Foreign Linear Algebra Textbooks

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ABSTRACT. Linear algebra is a basic subject for most engineering majors. The effect of learning has a very important influence on the future research and work for students, and the textbook is a very critical factor in the learning process. Therefore, it is of great significance to enhance the textbook development of engineering linear algebra in colleges and universities. The construction of teaching materials is of great significance. This paper first analyses the existing problems of domestic textbooks, and then introduces the experience and practices of excellent foreign textbooks. On this basis, it further analyses the reasons for the low quality of domestic textbooks and puts forward suggestions for rectification.

KEYWORDS: Linear algebra, Textbook development, Chinese and foreign comparison

1. Introduction

Linear algebra is a basic subject for most engineering majors such as mathematics, computer, electronics, communication, and control. It is no exaggeration to say that it will be difficult to follow in more in-depth professional course study and scientific research without a good foundation of linear algebra. Taking the current very popular deep learning as an example, many algorithms about deep learning are closely related to the knowledge of linear algebra, especially the technical details of the underlying implementation. If the foundation of linear algebra is not well laid, it will be difficult to understand those algorithms deeply, let alone to innovate. As a result, domestic colleges and universities basically take linear algebra as one of the compulsory courses in the initial stage of the university.

To learn a course well, learning materials are very important, and a good textbook can often achieve a multiplier effect with half the effort. For undergraduate students, they lack the ability to distinguish the quality of teaching materials, and most of them can only passively accept to use the textbooks selected by the university. According to statistics, the linear algebra textbook adopted by most universities in our country is the one published by Tongji University. Although a small number of colleges and universities have also used self-edited textbooks, they are similar in content and logic to the Tongji textbook. Therefore, it can be said that the Tongji version of the linear algebra textbook is very representative in China, which is the reason that this paper chooses it as the research object.

However, this set of textbooks has been increasingly questioned on the Internet recently, which has aroused the authors' attention and thinking. This paper first summarizes the main reasons why Tongji textbooks are questioned in the second section, and then introduces the advanced and mature methods of foreign textbooks in the third section, followed by the analysis of the common reasons for the low quality of domestic engineering mathematics textbooks in the fourth section. Finally, some suggestions and conclusions are given in the fifth and sixth sections, respectively.

2. Defeats in Domestic Linear Algebra Textbooks

2.1 Irrational Structure Arrangement

Many students who posted on the Internet pointed out that the most prominent problem with Tongji's "Linear Algebra" textbook is the structural confusion. More specifically, the first chapter starts with the "determinant" in the textbook, and this unprepared introduction method makes it difficult to understand the content for many freshmen students who have not learned the basic concepts of linear algebra.

If we take the geometry as a reference, the determinant is similar to the concept of volume. When a student understands points, lines, and surfaces, it is easy to generalize to volume, and the various formulas and properties of volume do not need to be memorized by rote. Similarly, if we first introduce the concepts of linear space, linear

transformation, matrix, etc., and then introduce the determinant, the process will be much smoother, so that we can also avoid using the inverted ordinal concept to define the determinant which is not a concept easy to explain.

The above is the layout logic of the determinant in Chapter 1, and the structure of subsequent chapters is similar in that textbook. From the beginning, the definition was introduced forcibly, and easy-to-understand geometric explanations were rarely given. As a result, students did not understand the use of this concept at all and why it is defined as shown in the formula, which makes it impossible to understand and memorize. To cope with homework and final exams, most students had to adopt a rote strategy.

2.2 Abstract Content and Lack of Cases

When teachers are in class and when compiling textbooks, they will explain or emphasize that the current theory has great application value in actual engineering, but there is often no actual case as a support, which to a certain extent obliterates students' interest and enthusiasm for learning. In fact, engineering mathematics is different from science mathematics, which should emphasize how to apply these mathematical tools instead of instilling pure mathematical concepts and deductions. Science can go from mathematics to mathematics, from formulas to formulas, but the education of engineering mathematics should emphasize the connection between mathematics and practical application problems. There are many applications of linear algebra, including many intuitive application examples in our lives, which can completely help explain abstract content concretely.

Another advantage of using application cases to interpret mathematics is that what you have learned is not easy to forget. The human brain has undergone a long-term evolutionary mechanism that makes us forget information or knowledge that the brain considers useless. You won't remember what time you got up on a certain day ten years ago, or what you ate for breakfast a year ago. But the addition, subtraction, multiplication, and division you learned when you were a kid is hard to forget. This is because our brains will automatically delete those useless information. Therefore, when we learn things from a young age and understand what we learn, it is very easy to remember, because when we understand, our brain will assume that those are useful, and then produce a deeper memory. If you learn mathematics simply by rote memorization, you may indeed be able to cope with the exam in a short time, but when you pass the exam, what you rote memorize loses its meaning and your brain thinks these things are useless, thus you may easily forget what you have learned after the exam.

3. Features and Advantages of Foreign Linear Algebra Textbooks

According to statistics from MIT's OCW official website, the "Linear Algebra" course released by Professor Gilbert Strang has more than 10 million visits since 2002, making it one of the most popular linear algebra courses in the world. Hence, this paper chooses it as a typical representative foreign textbook. Gilbert Strang's linear algebra textbook "Introduction to Linear Algebra" is popular for many reasons, which can be summarized in the following two points:

3.1 Practical and Moderate Difficulty

Most people who have studied that course said that Strang's textbook is more practical and of moderate difficulty. It pays more attention to cultivating mathematical intuition from practical problems and is more suitable for engineering students.

This is quite different from some domestic textbooks. The textbooks we usually use or read generally give definitions at first, followed by theorems and proof methods, which can easily make non-mathematics students lose interest. By contrast, Professor Strang's textbook first tells you some interesting mathematical facts, and then tells you how we solve the simpler problems (some methods even rely on trial and mathematical intuition), and then explores with you why this solution is right and whether there is a theoretical basis. After that, it leaves some exercises for yourself to validate that it is really right, and some other in-depth investigations are finally conducted which can be further refined into theorems.

3.2 Turn Abstract into Concrete

For people with poor mathematical foundations, linear algebra is really a very abstract course. But from the students' evaluation of Professor Strang's "Linear Algebra" textbook, it is considered consistently that the textbook is not so abstract and it can even be connected to high school. Professor Strang will insert many examples in his explanation of linear algebra, allowing students to understand some abstract concepts based on examples, which is very friendly to non-mathematics students. Some students said that they feel that many concepts are no longer rote.

4. Why is the Quality of Domestic Engineering Mathematics Textbooks Generally Low?

After expounding and comparing the advantages and disadvantages of linear algebra teaching materials at home and abroad, it is necessary for us to further think about the deeper problems under the surface. Isn't it possible that we cannot write good textbooks in China? The answer is obviously no, especially in recent years there have been many capable people in China, and there are many people who can write high-quality textbooks. Then why are the domestic engineering mathematics textbooks generally of low quality? The main reasons can be summarized as follows:

4.1 Market Pricing Cannot Match the Price of High-Quality Textbooks

To develop a high-quality textbook in a certain field, the cost (taking into account the hourly value of high-level authors and the time it takes) is extremely high. For example, Nobel Prize winner Mankiw wrote a classic textbook on the principles of economics, and the royalties can get up to tens of millions of dollars. It is known that if you buy genuine textbooks in foreign languages, it costs thousands of dollars. Because new books are very expensive, foreign students generally buy second-hand books or go to the library to borrow books or borrow books from seniors. This shows that a high-quality textbook is worth the price. The reason is also very simple, high quality will inevitably bring high prices. However, due to our long-standing disregard for intellectual property rights, we have long been accustomed to the recognition that a book should only cost dozens of dollars, which has led to the fact that people who can write high-quality textbooks cannot actually get the equivalent compensation they deserve.

4.2 Bad Money Drives out Good

Under the pressure of appraisal of professional titles, college teachers have to publish books in order to meet various selection criteria, and a considerable part of them are self-edited textbooks. In order to protect the interests of local teachers, or for other reasons, at present, a large number of domestic colleges and universities still use self-compiled textbooks, making high-quality textbooks unable to obtain a large number of orders through the market regulation mechanism. Therefore, the living space of high-quality textbooks was suppressed to a large extent, that is, bad money expelled good money.

5. Countermeasures and Suggestions

5.1 Reinforce the Incentive Mechanism of Textbook Development

Colleges and universities should comprehensively consider all aspects of textbook development, and further support and encourage textbook development in terms of time and funds. We should also increase investment in all aspects and further increase teachers' enthusiasm for compiling textbooks. At the same time, we should actively carry out the evaluation of textbooks, and give material or honorary rewards to teachers who have been struggling in the front line of textbook development for a long time.

5.2 Change the Concept of Textbook Compiling

The basic function of the textbook is to serve as an auxiliary tool, enabling teachers to smoothly transfer knowledge to students. In order to fully realize this function, it is necessary to provide a more acceptable way to strengthen the learning effect. According to investigations and studies, appropriate cases and guidance are of great help to increase students' enthusiasm and learning effect. Therefore, in the process of writing textbooks, we should consider issues from the perspective of students, instead of just list knowledge points.

5.3 Optimize Performance Evaluation System

At present, teachers in colleges and universities have a lot of pressure on performance appraisal in order to evaluate professional titles. They have to teach, publish papers, conduct scientific research, write patents, apply for funds, and write books and textbooks. Any shortcoming will become a promotion obstacle. This kind of rule requires people to become all-rounders, but in fact people are always particularly good at some aspects and have deficiencies in others. In order to satisfy this comprehensive performance appraisal system, most people choose to overcome their shortcomings, rather than vigorously promote their strengths. If things go on like this, it will consume and waste human resources. If the performance appraisal system can be optimized so that everyone can give full play to their strengths and focus on the careers they love, high-quality textbooks will surely be able to bloom everywhere in the motherland.

6. Conclusion

This paper takes linear algebra as an example to compare the advantages and disadvantages of Chinese and foreign engineering mathematics textbooks. At present, the main problems of Chinese textbooks are irrational structure, abstract content, and lack of cases. By contrast, foreign textbooks give people the impression of moderate difficulty, practicality, and ability to turn abstract into concrete. In addition to the superficial phenomenon, this paper also analyses the underlying reasons for the low quality of domestic textbooks, including copyright awareness and regional protection, and finally gives targeted countermeasures and suggestions based on the analysis.

Acknowledgement

This work is supported by the National Natural Science Foundation of China under grant no. 71901210.

References

- [1] Son, J.W., Hu, Q., “The initial treatment of the concept of function in the selected secondary school mathematics textbooks in the US and China”, *International Journal of Mathematical Education in Science and Technology*, Vol. 47, No. 4, pp. 505 – 530, 2016.
- [2] Glasnovic Gracin D. “Requirements in mathematics textbooks: a five-dimensional analysis of textbook exercises and examples”, *International journal of mathematical education in science and technology*, Vol. 49, No. 7, pp. 1003-1024, 2018.
- [3] Mei L. “Comparative Analysis of Some Linear Algebra Textbooks in China and America”, *The Guide of Science & Education*, No. 11, pp. 1-6, 2015.
- [4] Gilbert Strang. *Introduction to Linear Algebra*, Wellesley-Cambridge Press, Fourth Edition, 2009.
- [5] Tongji University Department of Mathematics. *Linear Algebra* (sixth edition). Higher Education Press, 2014.